

9

AREAS OF PARALLELOGRAMS AND TRIANGLES

Congruent Figures and

Figures on the Same Base and Between the Same Parallels

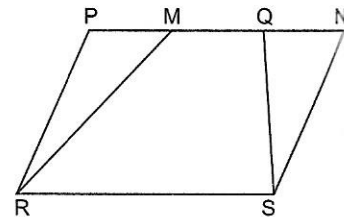
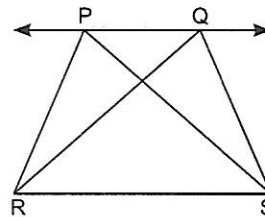
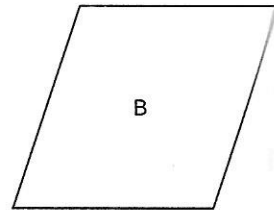
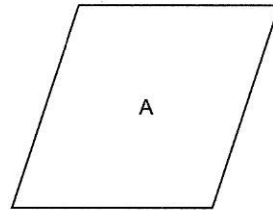
- Two figures are congruent, if they have the same shape and the same size.

In other words, if two figures A and B are congruent, then using a tracing paper, we can superpose one figure over the other such that it will cover the other completely.

So, if two figures A and B are congruent, they must have equal areas. But two figures having equal areas need not be congruent.

- Two figures are said to be on the same base and between the same parallels, if they have a common base (side) and the vertices (or the vertex) opposite to the common base of each figure lie on a line parallel to the base.

Note: It is noted that out of the two parallels, one must be the line containing the common base.



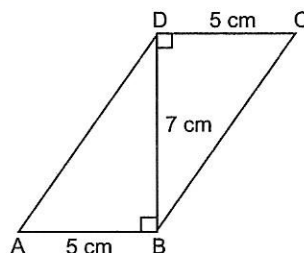
Parallelograms on the Same Base and Between the Same Parallels

- Parallelograms on the same base and between the same parallels are equal in area.
- If a triangle and a parallelogram are on the same base and between the same parallels, then the area of the triangle is equal to half the area of the parallelogram.

➤ SOLVED QUESTIONS BASED ON EXERCISES 9.1 AND 9.2

Very Short Answer Type Questions [1 Mark]

- In the given figure, ABCD is a parallelogram. Calculate the area of parallelogram ABCD.



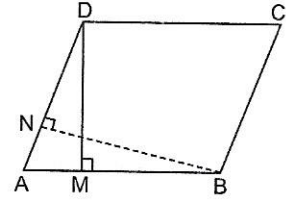
Sol. Area of parallelogram ABCD = base \times altitude = $AB \times DB = 5 \times 7 = 35 \text{ cm}^2$

2. Find the area of a rhombus, the length of whose diagonals are 16 cm and 12 cm respectively.

Sol. Area of rhombus = $\frac{1}{2} \times d_1 \times d_2 = \frac{1}{2} \times 16 \times 12 \text{ cm}^2 = 96 \text{ cm}^2$

3. In parallelogram ABCD, AB = 8 cm and the altitudes corresponding to sides AB and AD are DM = 6 cm and BN = 10 cm respectively. Find AD.

Sol. Area of parallelogram = base \times altitude
 $\therefore AD \times BN = AB \times DM$
 $AD \times 10 = 8 \times 6$
 $AD = \frac{8 \times 6}{10} = \frac{48}{10} = 4.8 \text{ cm}$



4. What will be the ratio of areas of the two parallelogram which lie on same base and between the same parallels?

Sol. 1 : 1

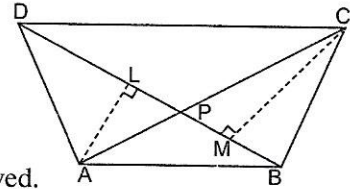
Short Answer Type Questions I [2 Marks]

5. Diagonals AC and BD of a quadrilateral ABCD intersect each other at P. Show that

$$\text{ar}(\triangle APB) \times \text{ar}(\triangle CPD) = \text{ar}(\triangle APD) \times \text{ar}(\triangle BPC)$$

Sol. We have

$$\begin{aligned} \text{ar}(\triangle APD) \times \text{ar}(\triangle BPC) &= \left(\frac{1}{2} \times AL \times DP\right) \times \left(\frac{1}{2} \times CM \times BP\right) \\ &= \left(\frac{1}{2} \times BP \times AL\right) \times \left(\frac{1}{2} \times DP \times CM\right) \\ &= \text{ar}(\triangle APB) \times \text{ar}(\triangle CPD) \text{ Hence proved.} \end{aligned}$$



6. If P is any point in the interior of a parallelogram ABCD, then prove that area of $(\triangle APB)$ is less than half the area of the parallelogram.

Sol. Given: P is any point in the interior of parallelogram ABCD.

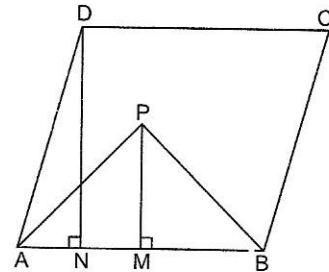
To prove: $\text{ar}(\triangle APB) < \frac{1}{2} (\text{ar} \parallel\text{gm ABCD})$

Construction: Draw $DN \perp AB$ and $PM \perp AB$.

Proof: $\text{ar}(\parallel\text{gm ABCD}) = AB \times DN$
 $\text{ar}(\triangle APB) = \frac{1}{2} (AB \times PM)$

Now, $PM < DN$
 $\Rightarrow AB \times PM < AB \times DN$
 $\Rightarrow \frac{1}{2} (AB \times PM) < \frac{1}{2} (AB \times DN)$

$\Rightarrow \text{ar}(\triangle APB) < \frac{1}{2} \text{ar}(\parallel\text{gm ABCD})$



Hence proved.

Short Answer Type Questions II [3 Marks]

7. ABCD is a parallelogram. E is a point on BA such that $BE = 2EA$ and F is a point on DC such that $DF = 2FC$. Prove that AECF is a parallelogram whose area is one-third of the area of parallelogram ABCD.

Sol. Given: A parallelogram ABCD. E is a point on BA s.t. $BE = 2EA$ and F is a point on DC such that $DF = 2FC$.

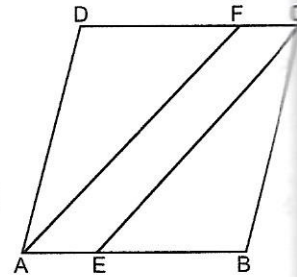
To prove: (i) AECF is a parallelogram.

(ii) $\text{ar}(\parallel\text{gm AECF}) = \frac{1}{3} \text{ar}(\parallel\text{gm ABCD})$

Proof:

⇒
⇒
⇒
⇒

$$\begin{aligned} BE &= 2EA \text{ and } DF = 2FC \\ AB - AE &= 2AE \text{ and } DC - FC = 2FC \\ AB &= 3AE \text{ and } DC = 3FC \\ AE &= \frac{1}{3} AB \text{ and } FC = \frac{1}{3} DC \\ AE &= FC \quad (\because AB = CD) \end{aligned}$$



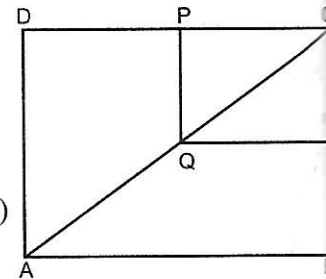
∴ AE || FC such that AE = FC
∴ AECF is a parallelogram.

Parallelograms ABCD and AECF have the same altitude and $AE = \frac{1}{3} AB$.

$$\therefore \text{ar}(\text{||gm AECF}) = \frac{1}{3} \text{ar}(\text{||gm ABCD})$$

Hence proved

8. ABCD and PQRC are rectangles. Q is the mid-point of AC. Show that P is the mid-point of DC and R is the mid-point of BC. Also, find the ratio of ar(ABCD) and ar(PQRC). [CBSE 2016]



Sol. Given: ABCD and PQRC are rectangles. Q is mid-point of AC.

To prove: P, R are mid-points of DC, BC respectively and find ar(ABCD) : ar(PQRC)

Proof: In ΔCAB , Q is the mid-point of AC.

$$QR \parallel AB$$

(∵ ABCD and PQRC both are rectangles)

⇒ R is the mid-point of BC.

(By converse of mid-point theorem)

Again in ΔCAB , Q and R are the mid-points of AC and BC respectively.

$$\Rightarrow QR = \frac{1}{2} AB$$

(By mid-point theorem)

$$\Rightarrow QR = \frac{1}{2} DC \quad \dots(i) \quad (\because AB = DC, \text{ opposite sides of a rectangle})$$

In ΔCAD , Q is the mid-point of AC.

Again, PQ || DA

⇒ P is the mid-point of DC.

(By converse of mid-point theorem)

Again in ΔCAD , Q and P are the mid-points of AC and DC respectively.

$$\Rightarrow PQ = \frac{1}{2} DA$$

(By mid-point theorem)

$$\Rightarrow PQ = \frac{1}{2} CB \quad \dots(ii) \quad (\because DA = CB, \text{ opposite sides of a rectangle})$$

Now,

$$\text{ar}(ABCD) = DC \times CB$$

$$\text{ar}(PQRC) = QR \times PQ$$

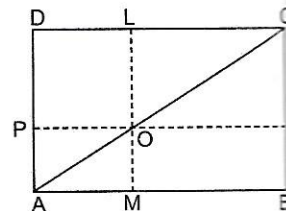
$$= \left(\frac{1}{2} DC\right) \times \left(\frac{1}{2} CB\right) = \frac{1}{4} DC \times CB = \frac{1}{4} (\text{ar } ABCD)$$

$$\Rightarrow \frac{\text{ar}(PQRC)}{\text{ar}(ABCD)} = \frac{1}{4} \text{ i.e. } = 1 : 4$$

Hence, ar (PQRC) : ar (ABCD) = 1 : 4

Long Answer Type Questions [4 Marks]

9. In the given figure, ABCD is a parallelogram. O is any point on AC. PQ || AB and LM || AD. Prove that ar(||gm DLOP) = ar(||gm BMOQ).



Sol. Given: ABCD is a parallelogram and point O lies on AC. PQ || AB and LM || AD.

To prove: ar (parallelogram DLOP) = ar (parallelogram BMOQ).

Proof: \therefore Diagonal of a parallelogram divides it into two triangles of equal areas

$$\begin{aligned} \therefore \quad & \text{ar}(\triangle ADC) = \text{ar}(\triangle ABC) \\ \Rightarrow \quad & \text{ar}(\triangle APO) + \text{ar}(\text{||gm DLOP}) + \text{ar}(\triangle OLC) = \text{ar}(\triangle AOM) \\ & \quad \quad \quad + \text{ar}(\text{||gm BMOQ}) + \text{ar}(\triangle OQC) \end{aligned} \quad \dots(i)$$

\therefore AO and OC are diagonals of parallelogram AMOP and OQCL respectively.

$$\therefore \quad \text{ar}(\triangle APO) = \text{ar}(\triangle AMO) \quad \dots(ii)$$

$$\text{ar}(\triangle OLC) = \text{ar}(\triangle OQC) \quad \dots(iii)$$

Subtracting (ii) and (iii) from (i), we get

$$\text{ar}(\text{||gm DLOP}) = \text{ar}(\text{||gm BMOQ}) \quad \text{Hence proved.}$$

10. The diagonals of a parallelogram ABCD intersect at a point O. Through O, a line is drawn to intersect AD at P and BC at Q. Show that PQ divides the parallelogram into two parts of equal area. [NCERT Exemplar]

Sol. **Given:** A parallelogram ABCD in which diagonals AC and BD intersect at O. Through O, a line is drawn to intersect AD at P and BC at Q.

To prove: $\text{ar}(\text{APQB}) = \text{ar}(\text{PQCD}) = \frac{1}{2} \text{ar}(\text{||gm ABCD})$

Proof: Diagonals of a parallelogram divides it into two triangles of equal area.

$$\begin{aligned} \text{ar}(\triangle ABC) &= \text{ar}(\triangle ACD) \\ \text{ar}(\text{ABQO}) + \text{ar}(\triangle COQ) &= \text{ar}(\text{CDPO}) + \text{ar}(\triangle AOP) \end{aligned} \quad \dots(i)$$

In $\triangle AOP$ and $\triangle COQ$, we have

$$\begin{aligned} \angle AOP &= \angle COQ && \text{(Vertically opposite angles)} \\ OA &= OC && \text{(Diagonal of a parallelogram bisect each other)} \\ \angle OAP &= \angle OCQ && \text{(Alternate interior angles)} \\ \therefore \quad \triangle AOP &\cong \triangle COQ && \text{(ASA congruence rule)} \end{aligned}$$

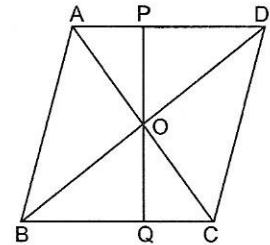
As congruent triangles are equal in areas,

$$\Rightarrow \quad \text{ar}(\triangle AOP) = \text{ar}(\triangle COQ) \quad \dots(ii)$$

From (i) and (ii), we get

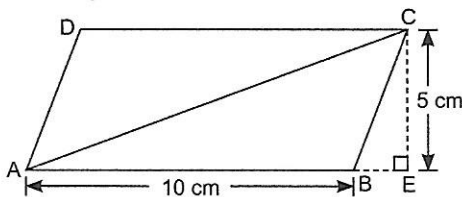
$$\begin{aligned} \text{ar}(\text{ABQO}) + \text{ar}(\triangle AOP) &= \text{ar}(\text{CDPO}) + \text{ar}(\triangle COQ) \\ \text{ar}(\text{ABQP}) &= \text{ar}(\text{CDPQ}) \\ \text{ar}(\text{APQB}) &= \text{ar}(\text{PQCD}) \end{aligned}$$

Hence proved.

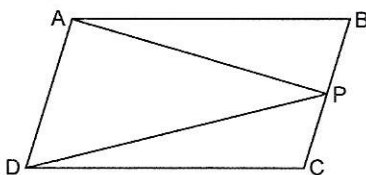


➤ PRACTICE QUESTIONS BASED ON EXERCISES 9.1 AND 9.2

1. In the given figure, compute the area of the parallelogram ABCD.

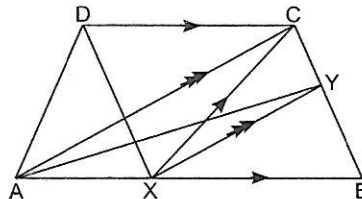


2. If area of parallelogram ABCD is 80 cm^2 , then find $\text{ar}(\triangle ADP)$.



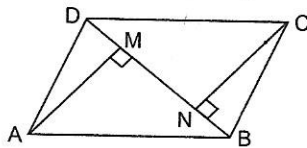
3. ABCD is a trapezium with $AB \parallel DC$. A line parallel to AC intersects AB at X and BC at Y.

Prove that $\text{ar}(\triangle ADX) = \text{ar}(\triangle ACY)$

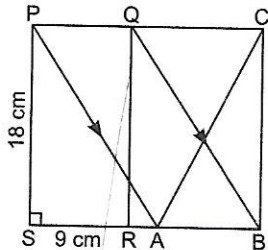


4. ABCD is a parallelogram. P is any point on CD. If $\text{ar}(\triangle DPA) = 18 \text{ cm}^2$ and $\text{ar}(\triangle APC) = 30 \text{ cm}^2$, find the area of $\triangle APB$.
5. BD is one of the diagonal of a quadrilateral ABCD. AM and CN are the perpendiculars from A and C respectively on BD. Show that

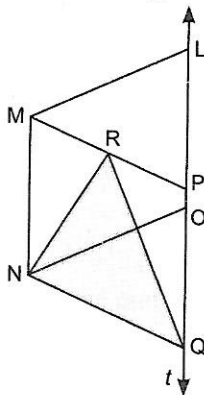
$$\text{ar (quadrilateral ABCD)} = \frac{1}{2} \times BD (AM + CN).$$



6. In a rectangle PQRS, PS = 18 cm and SR = 9 cm. Find area of $\triangle ABC$.

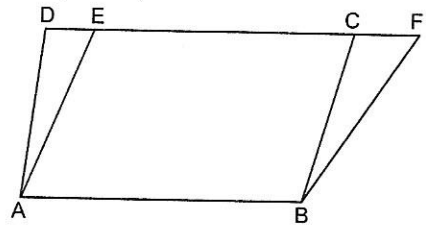


7. In the given figure, LMNO and PMNQ are two parallelograms. R is any point on side MP. Show that $\text{ar}(\triangle NRQ) = \frac{1}{2} \text{ar}(\text{||gm LMNO})$

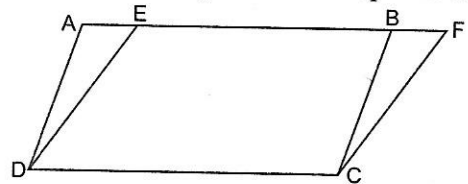


8. In the given figure, ABCD and ABFE are parallelograms and $\text{ar}(\text{quadrilateral EABC}) = 17 \text{ cm}^2$ and $\text{ar}(\text{parallelogram ABCD}) = 25 \text{ cm}^2$.

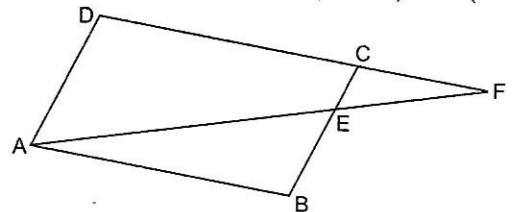
Find $\text{ar}(\triangle BCF)$.



9. Prove that the area of rhombus is half of the product of the diagonals.
10. If E, F, G, H are respectively the mid-points of the sides of a parallelogram ABCD and $\text{ar}(EFGH) = 40 \text{ cm}^2$, then find the value of $\text{ar}(ABCD)$.
11. ABCD is a trapezium in which $AB \parallel DC$, $DC = 30 \text{ cm}$ and $AB = 50 \text{ cm}$. If X and Y are respectively the mid-points of AD and BC, then prove that $\text{ar}(DCYX) = \frac{7}{9} \text{ar}(XYBA)$.
12. Prove that the parallelograms on the same base and between the same parallels are equal in area.



13. A point E is taken as the mid-point of the side BC of a parallelogram ABCD. AE and DC are produced to meet at F. Prove that $\text{ar}(\triangle ADF) = \text{ar}(ABFC)$



Triangles on the Same Base and between the Same Parallels

- Two triangles on the same base (or equal bases) and between the same parallels are equal in area.
- Two triangles having the same base (or equal bases) and equal areas lie between the same parallels.
- A median of a triangle divides it into two triangles of equal areas.

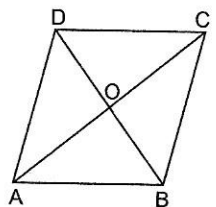
➤ SOLVED QUESTIONS BASED ON EXERCISE 9.3

Very Short Answer Type Questions [1 Mark]

1. In the given figure, ABCD is a parallelogram in which diagonals AC and BD intersect at O. If $\text{ar}(\text{||gm ABCD})$ is 68 cm^2 , then find $\text{ar}(\triangle OAB)$.

Sol. We have

$$\begin{aligned} \text{ar}(\triangle OAB) &= \frac{1}{4} \text{ar}(\text{||gm ABCD}) \\ &= \frac{1}{4} \times 68 \text{ cm}^2 = 17 \text{ cm}^2 \end{aligned}$$



2. ABC and BDE are two equilateral triangles such that D is the mid-point of BC. Prove that

$$\text{ar}(\triangle BDE) = \frac{1}{4} \text{ar}(\triangle ABC)$$

[NCERT Exemplar]

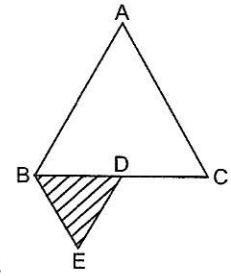
Sol. Let the side of triangle, $BC = a$

$$\Rightarrow BD = \frac{a}{2}$$

$$\begin{aligned} \therefore \text{ar}(\triangle BDE) &= \frac{\sqrt{3}}{4} \left(\frac{a}{2}\right)^2 \\ &= \frac{\sqrt{3}}{4} \times \frac{a^2}{4} = \frac{1}{4} \left(\frac{\sqrt{3}}{4} a^2\right) \end{aligned}$$

$$\therefore \text{ar}(\triangle BDE) = \frac{1}{4} \text{ar}(\triangle ABC)$$

Hence proved.



Short Answer Type Questions I [2 Marks]

3. D and E are the mid-points of BC and AD respectively of $\triangle ABC$. If area of $\triangle ABC = 20 \text{ cm}^2$, find area of $\triangle EBD$.

Sol. \because D is the mid-point of BC.

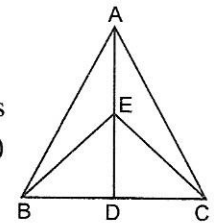
\therefore AD is the median of the $\triangle ABC$.

$$\Rightarrow \text{ar}(\triangle ABD) = \frac{1}{2} \text{ar}(\triangle ABC) \quad (\because \text{Median of a triangle divides it into two triangles of equal areas})$$

$$\Rightarrow \text{ar}(\triangle ABD) = \frac{1}{2} \times 20 \text{ cm}^2 = 10 \text{ cm}^2$$

Also, BE is the median of $\triangle ABD$.

$$\text{ar}(\triangle EBD) = \frac{1}{2} \text{ar}(\triangle ABD) = \frac{1}{2} \times 10 = 5 \text{ cm}^2$$



4. The medians BE and CF of a $\triangle ABC$ intersect at G. Prove that

$$\text{ar}(\triangle GBC) = \text{ar}(\triangle AFGE)$$

Sol. **Given:** Medians BE and CF of $\triangle ABC$ intersect at G.

To prove: $\text{ar}(\triangle GBC) = \text{ar}(\triangle AFGE)$

Proof: We have $\text{ar}(\triangle FBC) = \frac{1}{2} \text{ar}(\triangle ABC)$... (i)

(\because Median CF divides $\triangle ABC$ into two triangles of equal areas)

Similarly, $\text{ar}(\triangle EBC) = \frac{1}{2} \text{ar}(\triangle ABC)$... (ii)

From (i) and (ii), we get $\text{ar}(\triangle FBC) = \text{ar}(\triangle EBC)$... (iii)

Subtracting $\text{ar}(\triangle BGC)$ from both sides of (iii), we get

$$\text{ar}(\triangle FBC) - \text{ar}(\triangle BGC) = \text{ar}(\triangle EBC) - \text{ar}(\triangle BGC)$$

$$\Rightarrow \text{ar}(\triangle FGB) = \text{ar}(\triangle EGC)$$

Also, $\text{ar}(\triangle ABE) = \text{ar}(\triangle BEC)$... (iv)

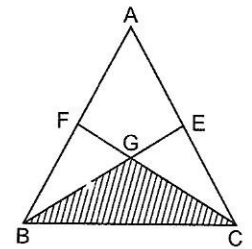
$$\Rightarrow \text{ar}(\triangle BFG) + \text{ar}(\triangle AFGE) = \text{ar}(\triangle BGC) + \text{ar}(\triangle GEC)$$

$$\Rightarrow \text{ar}(\triangle AFGE) = \text{ar}(\triangle BGC)$$

(\because BE is median of $\triangle ABC$)

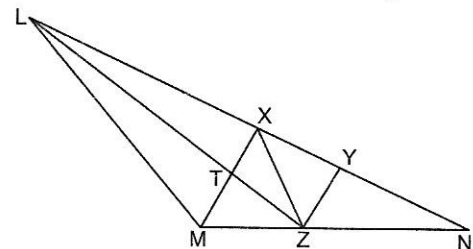
(Using (iv))

Hence proved.



5. In the given figure, X and Y are points on side LN of the $\triangle LMN$ such that $LX = XY = YN$. Through X, a line is drawn parallel to LM to meet MN at Z. Prove that $\text{ar}(\triangle LZY) = \text{ar}(\triangle MZYX)$

[NCERT Exemplar]



Sol. Given:

$$LX = XY = YN \text{ and } XZ \parallel LM$$

To prove:

$$\text{ar}(\Delta LZY) = \text{ar}(\Delta MZYX)$$

Proof:

$$XZ \parallel LM$$

ΔLXZ and ΔXMZ are on the same base XZ and between the same parallels XZ and LM .

$$\therefore \text{ar}(\Delta LXZ) = \text{ar}(\Delta XMZ)$$

Adding $\text{ar}(\Delta XYZ)$ on both sides, we get

$$\text{ar}(\Delta LXZ) + \text{ar}(\Delta XYZ) = \text{ar}(\Delta XMZ) + \text{ar}(\Delta XYZ)$$

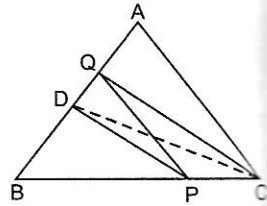
$$\therefore \text{ar}(\Delta LZY) = \text{ar}(\Delta MZYX)$$

Hence proved.

Short Answer Type Questions II [3 Marks]

6. In ΔABC , D is the mid-point of AB and P is any point on BC . If $CQ \parallel PD$ meets AB in Q in the given figure, then prove that $\text{ar}(\Delta BPQ) = \frac{1}{2} \text{ar}(\Delta ABC)$

[NCERT Exemplar]



Sol. Given: A ΔABC , D is the mid-point of AB and P is any point on BC and $CQ \parallel PD$ meets AB in Q .

To prove: $\text{ar}(\Delta BPQ) = \frac{1}{2} \text{ar}(\Delta ABC)$

Construction: Join D and C .

Proof: $\therefore CD$ is the median of ΔABC .

$$\therefore \text{ar}(\Delta BCD) = \frac{1}{2} \text{ar}(\Delta ABC) \quad (\because \text{Median of a triangle divides it into two triangles of equal areas}) \dots(i)$$

(Given)

$$\therefore \text{ar}(\Delta DPQ) = \text{ar}(\Delta DPC)$$

(Triangles are on the same base DP and between the same parallels DP and CQ)

Adding $\text{ar}(\Delta DBP)$ on both sides, we get

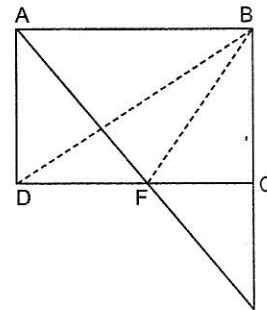
$$\text{ar}(\Delta DPQ) + \text{ar}(\Delta DBP) = \text{ar}(\Delta DPC) + \text{ar}(\Delta DBP)$$

$$\Rightarrow \text{ar}(\Delta BPQ) = \text{ar}(\Delta BCD) \dots(ii)$$

From (i) and (ii), we get $\text{ar}(\Delta PQB) = \frac{1}{2} \text{ar}(\Delta ABC)$

Hence proved.

7. In the given figure, $ABCD$ is a parallelogram in which BC is produced to E such that $CE = BC$. AE intersects CD at F . If area of $\Delta BDF = 3 \text{ cm}^2$, find the area of parallelogram $ABCD$. [NCERT Exemplar]



Sol. In ΔADF and ECF ,

$$\angle ADF = \angle ECF \quad (\text{Alternate interior angles})$$

$$AD = CE \quad (\because AD = BC \text{ and } BC = CE)$$

$$\angle DFA = \angle CFE \quad (\text{Vertically opposite angles})$$

$$\Delta ADF \cong \Delta ECF \quad (\text{AAS congruence rule})$$

\therefore

\Rightarrow

$$\text{ar}(\Delta ADF) = \text{ar}(\Delta ECF)$$

Also,

$$DF = CF \quad (\text{CPCT})$$

$\therefore F$ is the mid-point of DC .

$\Rightarrow BF$ is the median in ΔBCD

\Rightarrow

$$\text{ar}(\Delta BCD) = 2 \text{ar}(\Delta BDF)$$

(\because Median of a triangle divides it into two triangles of equal areas)

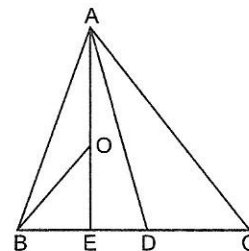
\Rightarrow

$$\text{ar}(\Delta BCD) = 2 \times 3 \text{ cm}^2 = 6 \text{ cm}^2$$

\therefore

$$\text{ar}(\text{||gm } ABCD) = 2 \text{ar}(\Delta BCD) = (2 \times 6) \text{ cm}^2 = 12 \text{ cm}^2$$

8. D is the mid-point of side BC of $\triangle ABC$ and E is the mid-point of BD. If O is the mid-point of AE, then prove that $\text{ar}(\triangle BOE) = \frac{1}{8} \text{ar}(\triangle ABC)$



Sol. Given: D, E and O are mid-points of BC, BD and AE respectively.

To prove: $\text{ar}(\triangle BOE) = \frac{1}{8} \text{ar}(\triangle ABC)$

Proof: \because AD and AE are the medians of $\triangle ABC$ and $\triangle ABD$ respectively.

$$\therefore \text{ar}(\triangle ABD) = \frac{1}{2} \text{ar}(\triangle ABC) \quad \dots(i)$$

and $\text{ar}(\triangle ABE) = \frac{1}{2} \text{ar}(\triangle ABD) \quad \dots(ii)$

Also, OB is the median of $\triangle ABE$.

$$\therefore \text{ar}(\triangle BOE) = \frac{1}{2} \text{ar}(\triangle ABE) \quad \dots(iii)$$

From (i), (ii) and (iii), we get

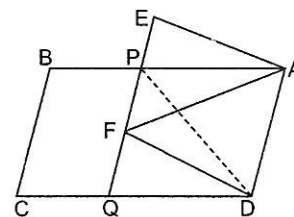
$$\begin{aligned} \text{ar}(\triangle BOE) &= \frac{1}{2} \text{ar}(\triangle ABE) = \frac{1}{2} \times \frac{1}{2} \text{ar}(\triangle ABD) \\ &= \frac{1}{4} \text{ar}(\triangle ABD) = \frac{1}{4} \times \frac{1}{2} \text{ar}(\triangle ABC) = \frac{1}{8} \text{ar}(\triangle ABC) \end{aligned}$$

Hence proved.

9. In the given figure, ABCD and AEFD are two parallelogram. Prove that

- $PE = FQ$
- $\text{ar}(\triangle PEA) = \text{ar}(\triangle QFD)$
- $\text{ar}(\triangle APE) : \text{ar}(\triangle PFA) = \text{ar}(\triangle QFD) : \text{ar}(\triangle PFD)$

[HOTS]



Sol. Given: ABCD and AEFD are two parallelograms.

To prove: (i) $PE = FQ$

(ii) $\text{ar}(\triangle PEA) = \text{ar}(\triangle QFD)$

(iii) $\text{ar}(\triangle APE) : \text{ar}(\triangle PFA) = \text{ar}(\triangle QFD) : \text{ar}(\triangle PFD)$

Proof: (i) In $\triangle APE$ and $\triangle DQF$,

$$\angle APE = \angle DQF \quad \text{(Corresponding angles)}$$

$$AE = DF \quad \text{(Opposite sides of a parallelogram)}$$

$$\angle AEP = \angle DFQ \quad \text{(Corresponding angles)}$$

$$\triangle APE \cong \triangle DQF \quad \text{(AAS congruence rule)}$$

$$\Rightarrow PE = QF \quad \text{(CPCT)}$$

(ii) $\text{ar}(\triangle PEA) = \text{ar}(\triangle QFD) \quad \dots(i) \quad (\because \text{Congruent triangles are equal in areas})$

(iii) $\triangle PFA$ and $\triangle PFD$ are on the same base PF and between the same parallels PQ and AD.

$$\therefore \text{ar}(\triangle PFA) = \text{ar}(\triangle PFD) \quad \dots(ii)$$

Dividing (i) by (ii), we get

$$\frac{\text{ar}(\triangle APE)}{\text{ar}(\triangle PFA)} = \frac{\text{ar}(\triangle QFD)}{\text{ar}(\triangle PFD)}$$

$$\Rightarrow \text{ar}(\triangle APE) : \text{ar}(\triangle PFA) = \text{ar}(\triangle QFD) : \text{ar}(\triangle PFD)$$

Hence proved

Long Answer Type Questions [4 Marks]

10. In the given figure, X and Y are the mid-points of AC and AB respectively, $QP \parallel BC$ and CYQ and BXP are straight lines. Prove that

$$\text{ar}(\triangle ABP) = \text{ar}(\triangle ACQ)$$

Sol. Given: X and Y are mid-points of AC and AB respectively, $QP \parallel BC$ and CYQ and BXP are straight lines.

To prove: $\text{ar}(\triangle ABP) = \text{ar}(\triangle ACQ)$

Proof: \therefore X and Y are the mid-points of AC and AB respectively.

\therefore By mid-point theorem, $XY \parallel BC$

Triangles BYC and BXC are on the same base BC and between the same parallel XY and BC.

$\therefore \text{ar}(\triangle BYC) = \text{ar}(\triangle BXC)$

Subtracting $\text{ar}(\triangle BOC)$ from both sides, we get

$$\text{ar}(\triangle BYC) - \text{ar}(\triangle BOC) = \text{ar}(\triangle BXC) - \text{ar}(\triangle BOC)$$

$\Rightarrow \text{ar}(\triangle BOY) = \text{ar}(\triangle COX)$

Adding $\text{ar}(\triangle XOY)$ on both sides, we get

$$\text{ar}(\triangle BOY) + \text{ar}(\triangle XOY) = \text{ar}(\triangle COX) + \text{ar}(\triangle XOY)$$

$\Rightarrow \text{ar}(\triangle BXY) = \text{ar}(\triangle CXY)$... (i)

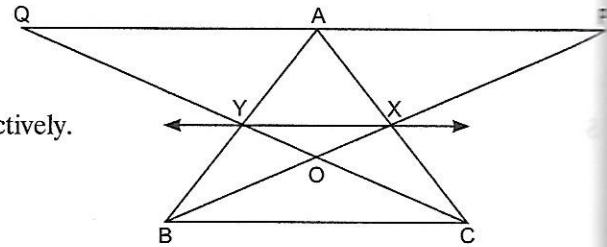
\therefore Parallelogram XYAP and Parallelogram XYAQ are on the same base XY and between the same parallels XY and PQ

$\therefore \text{ar}(\text{XYAP}) = \text{ar}(\text{XYQA})$... (ii)

Adding (i) and (ii), we get

$$\text{ar}(\triangle BXY) + \text{ar}(\text{XYAP}) = \text{ar}(\triangle CXY) + \text{ar}(\text{XYQA})$$

$\Rightarrow \text{ar}(\triangle ABP) = \text{ar}(\triangle ACQ)$



- 11. In the given figure, ABCDE is any pentagon. BP drawn parallel to AC meets DC produced at P and EQ drawn parallel to AD meets CD produced at Q. Prove that $\text{ar}(\text{ABCDE}) = \text{ar}(\triangle APQ)$ [NCERT Exemplar]**

Sol. Given: ABCDE is any pentagon, $BP \parallel AC$ and $EQ \parallel AD$

To prove: $\text{ar}(\text{ABCDE}) = \text{ar}(\triangle APQ)$

Proof: $\therefore \triangle ABC$ and $\triangle APC$ are on the same base AC and between the same parallels BP and AC

$\therefore \text{ar}(\triangle ABC) = \text{ar}(\triangle APC)$... (i)

Similarly, $\triangle AED$ and $\triangle AQD$ are on the same base AD and between the same parallels AD and EQ.

$\therefore \text{ar}(\triangle AED) = \text{ar}(\triangle AQD)$... (ii)

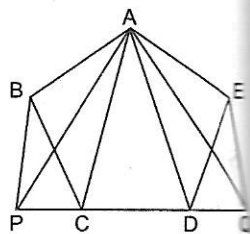
Adding (i) and (ii), we get

$$\text{ar}(\triangle ABC) + \text{ar}(\triangle AED) = \text{ar}(\triangle APC) + \text{ar}(\triangle AQD)$$

Adding $\text{ar}(\triangle ACD)$ on both sides of (iii), we get

$$\text{ar}(\triangle ABC) + \text{ar}(\triangle AED) + \text{ar}(\triangle ACD) = \text{ar}(\triangle APC) + \text{ar}(\triangle AQD) + \text{ar}(\triangle ACD)$$

$\Rightarrow \text{ar}(\text{ABCDE}) = \text{ar}(\triangle APQ)$... (iii)



Hence proved

- 12. If the medians of a $\triangle ABC$ intersect at G, show that $\text{ar}(\triangle AGC) = \text{ar}(\triangle AGB) = \text{ar}(\triangle BGC) = \frac{1}{3} \text{ar}(\triangle ABC)$. [NCERT Exemplar]**

Sol. Given: A $\triangle ABC$ in which medians AD, BE and CF intersect each other at G.

To prove: $\text{ar}(\triangle AGC) = \text{ar}(\triangle AGB) = \text{ar}(\triangle BGC) = \frac{1}{3} \text{ar}(\triangle ABC)$

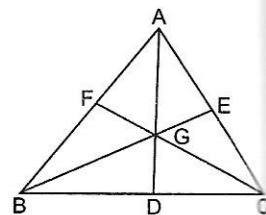
Proof: In $\triangle ABC$, AD is the median.

As a median of a triangle divides it into two triangles of equal areas,

$\therefore \text{ar}(\triangle ABD) = \text{ar}(\triangle ACD)$... (i)

In $\triangle BGC$, GD is the median.

$\therefore \text{ar}(\triangle GBD) = \text{ar}(\triangle GCD)$... (ii)



Hence proved

Subtracting (ii) from (i), we get

$$\begin{aligned} \text{ar}(\triangle ABD) - \text{ar}(\triangle GBD) &= \text{ar}(\triangle ACD) - \text{ar}(\triangle GCD) \\ \text{ar}(\triangle AGB) &= \text{ar}(\triangle AGC) \end{aligned} \quad \dots(iii)$$

Similarly,

$$\text{ar}(\triangle AGB) = \text{ar}(\triangle BGC) \quad \dots(iv)$$

From (iii) and (iv), we get

$$\text{ar}(\triangle AGB) = \text{ar}(\triangle BGC) = \text{ar}(\triangle AGC) \quad \dots(v)$$

But $\text{ar}(\triangle AGB) + \text{ar}(\triangle BGC) + \text{ar}(\triangle AGC) = \text{ar}(\triangle ABC)$

...

From (v) and (vi), we get

$$3\text{ar}(\triangle AGB) = \text{ar}(\triangle ABC)$$

$$\text{ar}(\triangle AGB) = \frac{1}{3}\text{ar}(\triangle ABC)$$

Hence,

$$\text{ar}(\triangle AGB) = \text{ar}(\triangle AGC) = \text{ar}(\triangle BGC) = \frac{1}{3}\text{ar}(\triangle ABC)$$

13. In the given figure, ABCD is a parallelogram. Prove that $\text{ar}(\triangle BCP) = \text{ar}(\triangle DPQ)$, if $BC = CQ$.

Sol. Given: A parallelogram ABCD in which $BC = CQ$.

To prove: $\text{ar}(\triangle BCP) = \text{ar}(\triangle DPQ)$

Construction: Join AC.

Proof: $\because \triangle APC$ and $\triangle BPC$ are on the same base PC and between the same parallels PC and AB.

$$\therefore \text{ar}(\triangle APC) = \text{ar}(\triangle BPC)$$

Since ABCD is a parallelogram, $AD = BC$

$$\Rightarrow AD = CQ$$

Now, $AD \parallel CQ$ and $AD = CQ$

Thus, in quadrilateral ADQC, one pair of opposite sides is equal and parallel.

\therefore ADCQ is a parallelogram.

$\Rightarrow AP = PQ$ and $CP = DP$ (Diagonals of a parallelogram bisect each other)

Now, in $\triangle APC$ and $\triangle DPQ$, $AP = PQ$ (Proved above)

$\angle APC = \angle DPQ$ (Vertically opposite angles)

$PC = PD$ (Proved above)

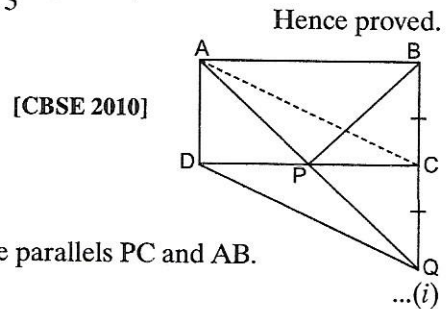
$\therefore \triangle APC \cong \triangle DPQ$ (SAS congruence rule)

$\Rightarrow \text{ar}(\triangle APC) = \text{ar}(\triangle DPQ)$... (i)

From (i) and (ii), we get

$$\text{ar}(\triangle BCP) = \text{ar}(\triangle DPQ) \quad \dots(ii)$$

Hence proved.

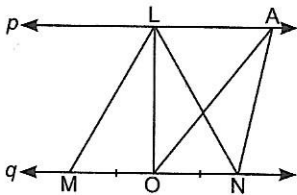


[CBSE 2010]

PRACTICE QUESTIONS BASED ON EXERCISE 9.3

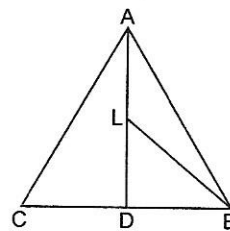
1. Show that a median of a triangle divides it into two triangles of equal areas.

2. In the given figure, $p \parallel q$ and O is the mid-point of MN. If $\text{ar}(\triangle AON) = x \text{ar}(\triangle LMN)$, then find the value of x.

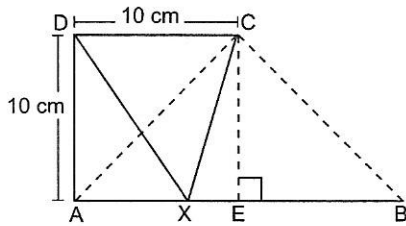


3. In the given figure, D is the mid-point of BC and L is the mid-point of AD. Prove that

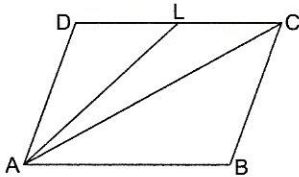
$$\text{ar}(\triangle ABL) = \frac{1}{4} \text{ar}(\triangle ABC)$$



4. In the given figure, $AB \parallel DC$. Find the area of triangle DCX.

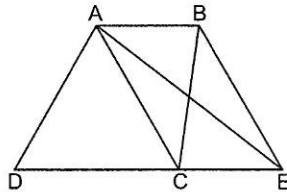


5. In the given figure, ABCD is a parallelogram and L is the mid-point of DC. If $\text{ar}(\triangle ACL) = 72 \text{ cm}^2$, then find $\text{ar}(\triangle ADC)$.



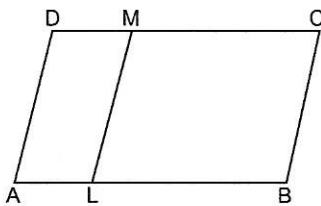
6. $\triangle ABC$ and $\triangle DBC$ are on the same base BC with vertices A and D on opposite side of BC such that area of $\triangle ABC = \text{area of } \triangle DBC$. Show that BC bisects AD.
7. In $\triangle ABC$, if D is a point on BC and divides it in the ratio 3 : 5, i.e. if $BD : DC = 3 : 5$, then find the ratio of $\text{ar}(\triangle ADC)$ and $\text{ar}(\triangle ABC)$.

8. In the given figure, ABCD is a quadrilateral and $BE \parallel AC$ and also, BE meets DC produced at E. Show that area of $\triangle ADE$ is equal to the area of the quadrilateral ABCD.

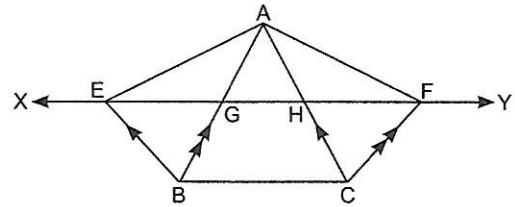


9. In a parallelogram ABCD, if ML divides AL and LB in the ratio 1 : 2, then prove that

$$\text{ar}(\parallel \text{gm ALMD}) = \frac{1}{2} \text{ar}(\parallel \text{gm LBCM})$$

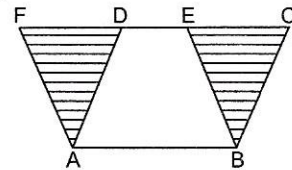


10. XY is a line parallel to side BC of a $\triangle ABC$. If $BE \parallel AC$ and $CF \parallel AB$ meets XY at E and F respectively, show that $\text{ar}(\triangle ABE) = \text{ar}(\triangle ACF)$.



Value Based Questions

- A bank gets a piece of land on lease by government in the heart of city to promote its commercial advertisement. If the piece of land is in parallelogram shape of area 120 m^2 and if equal area is divided by Bank Authority for cleaning atmosphere and commercial activity, then
 - Evaluate the area of two equal parts.
 - Which values are being promoted by the bank through this activity ?
- A farmer has two pieces of land in a parallelogram shape on the same base. He grows vegetables and flowers in two triangular pieces of land as shown in diagram:

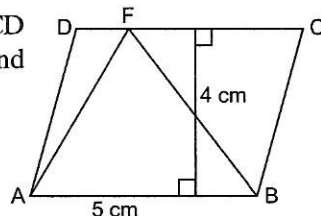


- Check whether the two triangular pieces of land are equal in areas.
 - What values are being displayed by the farmer?
- On a square plot of land, a farmer wants to grow five different crops at a time. On half area, he wants to grow wheat. But in rest four equal parts, he wants to grow four different crops.
 - How can he divide the area, suggest diagrammatically?
 - Which type of crop pattern is the farmer following ?

INTEGRATED EXERCISE

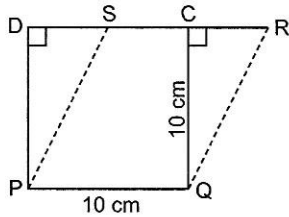
Very Short Answer Type Questions [1 Mark]

1. In the given figure, ABCD is a parallelogram. Find $\text{ar}(\triangle AFB)$.



- The area of a parallelogram ABCD is 36 cm^2 . What is the area of $\triangle ABC$?
- If a triangle and a parallelogram are on the same base and between the same parallels, then find the ratio of the area of the triangle to the area of parallelogram.

4. In the given figure, PQRS is a parallelogram and PQCD is a square. Find $\text{ar}(PQRS)$.



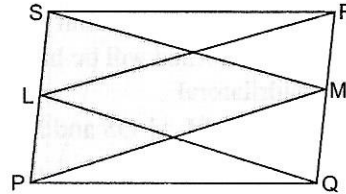
9. If a triangle and a parallelogram are on the same base and between the same parallels, then prove that the area of the triangle is equal to half the area of the parallelogram.

Short Answer Type Questions II [3 Marks]

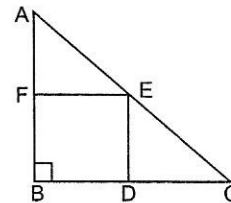
10. In the given figure, PQRS is a parallelogram. L is any point on PS and M is any point on QR.

Show that:

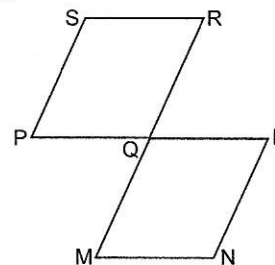
- (i) $\text{ar}(\triangle PMS) = \text{ar}(\triangle PLQ) + \text{ar}(\triangle LRS)$
(ii) $\text{ar}(\triangle QLR) = \text{ar}(\triangle PQM) + \text{ar}(\triangle SRM)$



11. BDEF is a square formed inside an isosceles right triangle ABC, right-angled at B. Show that $\text{ar}(\triangle AFE) = \text{ar}(\triangle CDE)$.



12. The sides PQ and RQ of a parallelogram PQRS are produced to L and M respectively such that $PQ = QL$ and $RQ = QM$ to form another parallelogram QLMN.



Prove that $\text{ar}(\text{|| gm PQRS}) = \text{ar}(\text{|| gm QLMN})$

13. A point D is taken on the side BC of a $\triangle ABC$ such that $BD = 2DC$. Prove that $\text{ar}(\triangle ABD) = 2\text{ar}(\triangle ADC)$.

14. ABCD is a parallelogram whose diagonals intersect at O. If P is any point on BO, prove that

- (i) $\text{ar}(\triangle ADO) = \text{ar}(\triangle CDO)$
(ii) $\text{ar}(\triangle ABP) = \text{ar}(\triangle CBP)$

Short Answer Type Questions I [2 Marks]

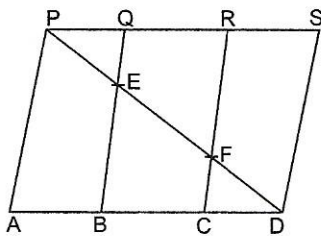
5. ABC and BDE are two equilateral triangles such that D is the mid-point of BC. Then

$$\text{ar}(BDE) = 4 \text{ar}(\triangle ABC)$$

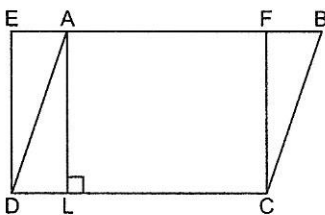
Write true or false and justify your answer.

6. In the given figure, PSDA is a parallelogram. Points Q and R are taken on PS such that $PQ = QR = RS$ and $PA \parallel QB \parallel RC$. Prove that $\text{ar}(\triangle PQE) = \text{ar}(\triangle CDF)$.

[NCERT Exemplar]



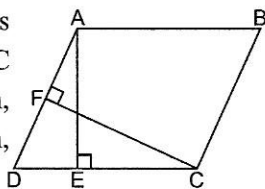
7. In the given figure, ABCD is a parallelogram and EFCB is a rectangle.



Also, $AL \perp DC$. Prove that

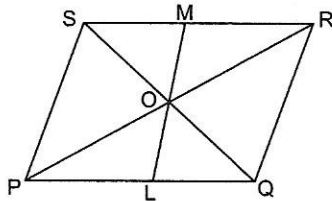
- (i) $\text{ar}(ABCD) = \text{ar}(EFCB)$
(ii) $\text{ar}(ABCD) = DC \times AL$

8. In the given figure, ABCD is a parallelogram. $AE \perp DC$ and $CF \perp AD$. If $AB = 8$ cm, $AE = 4$ cm and $CF = 5$ cm, then find AD.



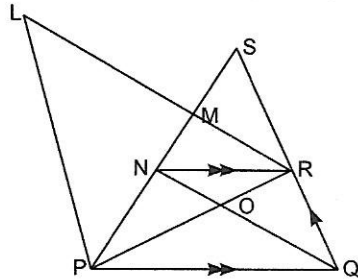
Long Answer Type Questions [4 Marks]

15. The diagonals of a parallelogram PQRS intersect each other in O. A line through O meets PQ in L and RS in M. Show that $\text{ar}(\text{PLMS}) = \frac{1}{2} \text{ar}(\text{||gm PQRS})$



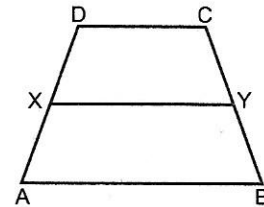
16. If the mid-points of the sides of a quadrilateral are joined in order, prove that the area of the parallelogram so formed will be half of the area of the given quadrilateral.
17. In the given figure, $PL \parallel QS$ and $NR \parallel PO$.

- (i) Name the triangle equal in area of ΔRQO .
 (ii) Prove that $\text{ar}(\Delta LMS) = \text{ar}(\Delta RMP)$
 (iii) Prove that $\text{ar}(\text{MNQR}) = \text{ar}(\Delta LMS)$



18. In the given figure, ABCD is a trapezium in which $AB \parallel DC$, $DC = 40$ cm and $AB = 60$ cm. If X and Y are respectively the mid-points of AD and BC, prove that

- (i) $XY = 50$ cm
 (ii) DCYX is a trapezium



19. ABCD is a parallelogram. X and Y are the mid-points of BC and CD respectively.

Prove that $\text{ar}(\Delta AXY) = \frac{3}{8} \text{ar}(\text{||gm ABCD})$.

[HOTS]

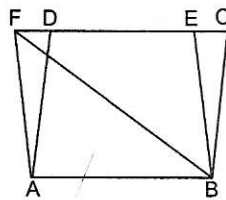
20. ABCD is a parallelogram and X is mid-point of AB. If $\text{ar}(\text{AXCD}) = 24 \text{ cm}^2$, then $\text{ar}(\Delta ABC) = 24 \text{ cm}^2$. Write true or false and justify your answer.

[NCERT Exemplar]

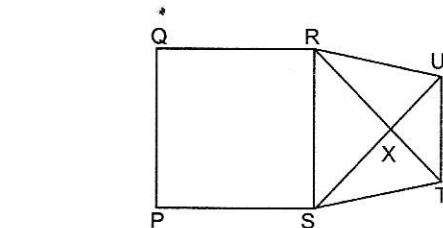
ASSESS YOURSELF

1. The area of a parallelogram ABCD is 90 cm^2 . Find

- (i) $\text{ar}(\text{||gm ABEF})$
 (ii) $\text{ar}(\Delta ABD)$
 (iii) $\text{ar}(\Delta BEF)$

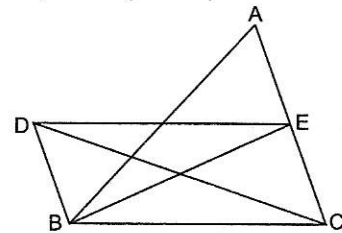


2. The diagonal of a square is 8 cm. What is its area?
 3. The area of a trapezium is 60 cm^2 . The distance between its parallel sides is 6 cm. If one of the parallel sides is 8 cm, then find the other parallel side.
 4. In ΔABC , X and Y are points on sides AB and AC respectively. If $\angle YXC = \angle BCX$, prove that $\text{ar}(\Delta BXC) = \text{ar}(\Delta BYC)$. [CBSE 2016]
 5. PQRTS is a pentagon. A line through T meets QR produced in U such that $SR \parallel UT$. Show that $\text{ar}(\text{PSUQ}) = \text{ar}(\text{PQRTS})$. [CBSE 2015]



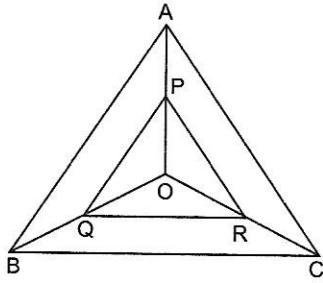
6. In the given figure, $BD \parallel CA$, E is the mid-point of CA and $BD = \frac{1}{2} CA$. Prove that

$$\text{ar}(\Delta ABC) = 2\text{ar}(\Delta DBC).$$

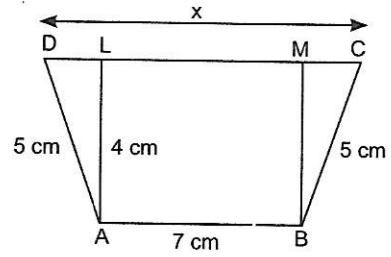


7. P and Q are points on side BC of ΔABC such that P and Q intersect BC. Show that $\text{ar}(\Delta PQA) = \text{ar}(\Delta PAB)$. [CBSE 2016]

8. ABC is an equilateral triangle with perimeter 30 cm. P, Q and R are mid-points of AO, BO and CO respectively. Find $\text{ar}(\Delta PQR)$. [CBSE 2016]



9. In the given figure, ABCD is a trapezium in which $AB = 7$ cm, $AD = BC = 5$ cm, $DC = x$ cm and distance between AB and DC is 4 cm. Find the value of x and area of trapezium ABCD.



10. A parallelogram and a rectangle have common base and equal areas. Show that the perimeter of the rectangle is smaller than the perimeter of the parallelogram.

